9709|MAY|2026|32 (Leaked)

① Solve $\ln x + \ln(2x+1) = 2\ln(x+1)$

$$\cancel{\ln}(x(2x+1)) = \cancel{\ln}(x+1)^2$$

$$2x^2+x = x^2+2x+1 \Rightarrow x^2-x-1=0.$$

$$x = \frac{1 \pm \sqrt{1^2-4(-1)(1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$x>0 \therefore \quad x = \frac{1+\sqrt{5}}{2} = 1.62$$ [4]

③ Coef of x^3 in expansion of $(5-3x)\sqrt{1+2x}$.

(a) $$(5-3x)(1+2x)^{1/2}$$

$$(5-3x)\left(1+\frac{1}{2}(2x)+\frac{\frac{1}{2}(-\frac{1}{2})(2x)^2}{2!}+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(2x)^3}{3!}\right)$$

$$(5-3x)\left(1+x-\frac{1}{2}x^2+\frac{1}{2}x^3\right)$$

$$\frac{3}{2}x^3+\frac{5}{2}x^3 = 4x^3 \therefore \text{ coef is } \underline{4}.$$ [4]

(b) Valid for $|2x|<1 \therefore |x|<1/2$
Consider $(1+2x)^{1/2}$ since $(5-3x)$ valid for all real x.
[1]

④ $y = \dfrac{e^{-2x}\cos x}{3+2\sin x}$ Find eqn of tangent at $x=0$. [5]

when $x=0$; $y = \dfrac{e^0\cos 0}{3+2\sin 0} = \dfrac{1}{3}$.

$y=\dfrac{u}{v}$ $\quad u = e^{-2x}\cos x$:

$$u' = \cos x(-2e^{-2x}) + e^{-2x}(-\sin x)$$
$$= -2e^{-2x}\cos x - e^{-2x}\sin x.$$

$$y' = \frac{(3+2\sin x)(u') - (e^{-2x}\cos x)(2\cos x)}{(3+2\sin x)^2};$$

when $x=0$; $y' = \dfrac{3(-2)-(1)(2)}{9} = -\dfrac{8}{9}. = m_{tangent}.$

2 The parametric equations of a curve are

$$x = 3e^{2t}, \qquad y = (5 - e^t)^3.$$

Find the exact gradient of the curve at the point where $t = \ln 2$, simplifying your answer. [4]

$$\frac{dx}{dt} = 3 \cdot 2e^{2t} = 6e^{2t}$$

$$\frac{dy}{dt} = 3(5-e^t)^2 \cdot (-e^t)$$

$$= -3e^t(5-e^t)^2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{-3e^t(5-e^t)^2}{6e^{2t}} = \frac{-(5-e^t)^2}{2e^{2t}}$$

dy/dx at $t = \ln 2$

$$dy/dx = -\frac{(5-e^{\ln 2})^2}{2e^{2\ln 2}}$$

$$= -\frac{(3)^2}{4}$$

$$\frac{dy}{dx} = -\frac{9}{4}$$

4 cont'd. $y - \frac{1}{3} = -\frac{8}{9}(x - 0)$

$y - \frac{1}{3} = -\frac{8}{9}x$.

$y + \frac{8}{9}x - \frac{1}{3} = 0$; $\times 9 \rightarrow 9y + 8x - 3 = 0$.

⑤ Solve $\tan(\theta + \frac{\pi}{4}) + 3\cot\theta + 5 = 0$ for $-\pi < \theta < \pi$. [6]

$$\frac{\tan\theta + 1}{1 - \tan\theta} + \frac{3}{\tan\theta} + \frac{5}{1} = 0.$$

$$\frac{\tan\theta(\tan\theta+1) + 3(1-\tan\theta) + 5(1-\tan\theta)(\tan\theta)}{(1-\tan\theta)(\tan\theta)(1)} = 0.$$

$\tan^2\theta + \tan\theta + 3 - 3\tan\theta + 5\tan\theta - 5\tan^2\theta = 0$.

$-4\tan^2\theta + 3\tan\theta + 3 = 0$.

$4\tan^2\theta - 3\tan\theta - 3 = 0$.

Let $u = \tan\theta$

$4u^2 - 3u - 3 = 0$.

apply quadratic formula.

$\Rightarrow u = 1.319$

$u = -0.569$.

$\tan\theta = 1.319$

$\theta = \tan^{-1}(1.319) = 0.922^c$ (1^{st} quadrant)

$0.922 - \pi = -2.22$

$\tan\theta = -0.569$ ∴ general angle $= 0.517$

in 2nd quad: ~~0.517~~ $\pi - 0.517 = 2.62$

in 4^{th} quad: $2\pi - 0.517 = 5.766 - 2\pi$

$= -0.517$.

$x = -2.22, -0.517, 2.62, 0.922$

⑥(a) $8\cos^4 x \equiv 3 + \cos 4x + 4\cos 2x$. [3]

$\cos^4 x = (\cos^2 x)^2$ from $\cos 2x = 2\cos^2 x - 1$.

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\Rightarrow \left(\frac{1}{2}\cos 2x + \frac{1}{2}\right)^2$

$= \left(\frac{1}{4}\cos^2 2x + \frac{1}{2}\cos 2x + \frac{1}{4}\right) \times 8 \Rightarrow \underbrace{2\cos^2 2x}_{} + 4\cos 2x + 2$.

since $\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$

$2\left(\frac{1}{2}\cos 4x + \frac{1}{2}\right) + 4\cos 2x + 2$

$\cos^2 2x = \frac{1}{2}\cos 4x + \frac{1}{2}$

$= \cos 4x + 1 + 4\cos 2x + 2$

$= 3 + \cos 4x + 4\cos 2x$.

$\varnothing$.

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 8\cos^4 x \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (3 + \cos 4x + 4\cos 2x) \, dx$.

$= 3x + \frac{1}{4}\sin 4x + 2\sin 2x \Big|_{\pi/6}^{\pi/3}$.

$= \pi + \left(-\frac{\sqrt{3}}{8}\right) + \sqrt{3} - \left(\frac{3\pi}{6} + \frac{\sqrt{3}}{8} + \sqrt{3}\right)$

$4\pi - \frac{\sqrt{3}}{8} + \sqrt{3} - \frac{\pi}{2} - \frac{\sqrt{3}}{8} - \sqrt{3}$

$= \frac{\pi}{2} - \frac{2\sqrt{3}}{8} \Rightarrow \frac{\pi}{2} - \frac{\sqrt{3}}{4}$

⑦(a) $\vec{OA} = 2i - 3j + 4k$ (a) exact value of cos of $\angle AOB$.

$\vec{OB} = 3i + j - 2k$.

$OA \cdot OB = |OA||OB|\cos\theta$.

$\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = 6 - 3 - 8 = -5$.

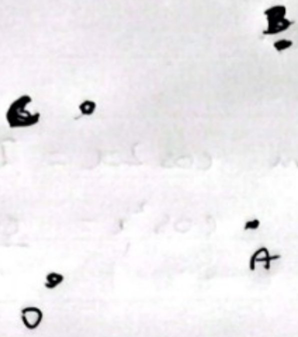
$|OA| = \sqrt{29}$

$|OB| = \sqrt{14}$

$\cos\theta = \frac{-5}{\sqrt{29}\sqrt{14}} = \frac{-5}{\sqrt{406}}$.

[3]

(b) OABC is a parallelogram. Find position vector of C. [2].



$\overrightarrow{BC} = \overrightarrow{AO}$

$\overrightarrow{BO} + \overrightarrow{OC} = \overrightarrow{AO}$

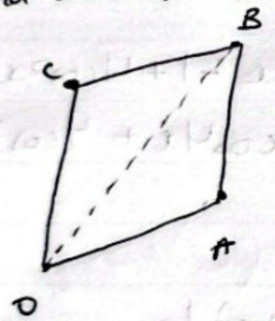
$\overrightarrow{OC} = \overrightarrow{AO} - \overrightarrow{BO}$

$= \begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -6 \end{pmatrix}$

$\overrightarrow{OC} = i + 4j - 6k$.

OR $\overrightarrow{OC} = \overrightarrow{AB}$

(c) Find area of OABC. [3].



Find area of triangle OAB and $\times 2$

Area $= \frac{1}{2}ab\sin C = \frac{1}{2}(OA)(OB)\sin AOB$.

$\cos AOB = \frac{-5}{\sqrt{406}}$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \frac{25}{406} = \frac{381}{406}$

$\sin\theta = \sqrt{\frac{381}{406}}$

or use triangles to find sin

$\frac{1}{2}\sqrt{29}\sqrt{14} \times \sqrt{\frac{381}{406}} \times 2$

$= \sqrt{406} \times \frac{\sqrt{381}}{\sqrt{406}} = \sqrt{381}$ units^2

(8) Solve $2(2+\sin y)\sin 2x = (1+2\cos 2x)\cos y\left(\frac{dy}{dx}\right)$

$y = \pi/2$ when $x = \pi/4$.

$\Rightarrow$ Seperate variables: $\frac{2\sin 2x}{1+2\cos 2x}\,dx = \frac{\cos y}{2+\sin y}\,dy$.

$\int$ bs.

$$-\frac{1}{2}\ln(1+2\cos 2x) = \ln(2+\sin y) + c.$$

$$-\frac{1}{2}\ln(1) = \ln 3 + c \quad \therefore \quad c = -\ln 3.$$

$$-\frac{1}{2}\ln(1+2\cos 2x) = \ln(2+\sin y) - \ln 3. \qquad \times -2.$$

$$\ln(1+2\cos 2x) + 2\ln(2+\sin y) - \ln 9. = 0.$$

$$\ln\left(\frac{(1+2\cos 2x)(2+\sin y)^2}{9}\right) = 0.$$

$$(1+2\cos 2x)(2+\sin y)^2 = 9.$$

when $x = \frac{\pi}{6}$;
$$2(2+\sin y)^2 = 9$$

$$(2+\sin y)^2 = \frac{9}{2}$$

$$2+\sin y = \sqrt{\frac{9}{2}} = \pm\frac{3}{\sqrt{2}}$$

$$\sin y = \frac{\pm 3}{\sqrt{2}} - 2 = \frac{-4+3\sqrt{2}}{2}, \underbrace{-\frac{4+3\sqrt{2}}{2}}_{\text{NO SOLN}}$$

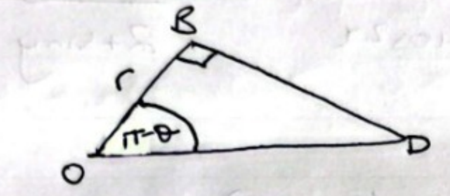
$$y = 0.122^c; \quad y = \pi - 0.122^c = 3.02^c.$$

(9) (a) Area of shaded region = Sector Area − triangle area

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

Consider triangle OBD.



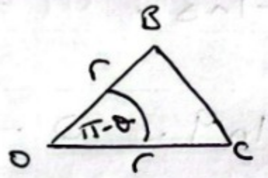
~~BD~~

$$\tan(\pi-\theta) = \frac{BD}{r}$$

$$\therefore BD = r\tan(\pi-\theta)$$

Area of OBD $= \frac{1}{2}r^2\tan(\pi-\theta)$

Consider triangle OBC.



Area $= \frac{1}{2}r^2\sin(\pi-\theta)$

recall $\sin(\pi-\theta) = \sin\theta$

$= \frac{1}{2}r^2\sin\theta$

$\therefore$ Area of BCD $= \frac{1}{2}r^2\tan(\pi-\theta) - \frac{1}{2}r^2\sin\theta$

$$\Rightarrow \cancel{\frac{1}{2}r^2}\left[\tan(\pi-\theta) - \sin\theta\right] = \frac{1}{2}\left[\cancel{\frac{1}{2}r^2}(\theta - \sin\theta)\right]$$

$$2\tan(\pi-\theta) = \theta - \sin\theta + 2\sin\theta$$

$$= \theta + \sin\theta$$

Ø [4]

(b) $2\tan(\pi-\theta) - \theta - \sin\theta = 0$.

$2.1 \to 0.456$
$2.2 \to -0.261$ } sign change $\therefore$ root lies in this interval.

(c) $\theta_{n+1} = \pi - \tan^{-1}\left(\frac{\theta_n + \sin\theta_n}{2}\right)$

$\theta_2 = 2.15$;

$\theta_3 = 2.16082$

$\theta_4 = 2.16007$

$\theta_5 = 2.16012$ $\theta \approx 2.160$.

$\theta_6 = 2.16012$

10(a)

$$\begin{array}{r|l} & x - 2 \quad \leftarrow \text{quotient} \\ x+2 & x^2+0x+0 \\ & -\ x^2+2x \downarrow \\ & \underline{0} -2x+0 \\ & -2x-4 \\ & \quad 4 \quad \leftarrow \text{remainder} \end{array}$$

[2]

(b) $\displaystyle\int_0^4 x\ln(x+2)\,dx$. [7]

$u=\ln(x+2) \qquad dv = x$

$du = \dfrac{1}{x+2} \qquad v = x^2/2$

$$\frac{x^2}{2}\ln(x+2)\Big|_0^4 - \underbrace{\frac{1}{2}\int\frac{x^2}{x+2}\,dx}_{\text{from part (i)}}$$

$$-\frac{1}{2}\int\left(x-2+\frac{4}{x+2}\right)dx$$

$$8\ln 6 - \frac{1}{2}\left[\frac{x^2}{2}-2x+4\ln(x+2)\right]_0^4$$

$$-\frac{1}{2}\Big[(8-8+4\ln 6)-(4\ln 2)\Big]$$

$$8\ln 6 - \frac{1}{2}(4\ln 6 - 4\ln 2)$$

$8\ln 6 - 2\ln 6 - 2\ln 2 \Rightarrow 6\ln 6 + 2\ln 2$

$6\ln(3\times 2)+2\ln 2$

$6\ln 3 + 6\ln 2 + 2\ln 2$

$6\ln 3 + 8\ln 2$

11.) $p(z) = 2z^4 - 3z^3 - 4z^2 + z + 30$.

[3]

$z = 2-i$ is a root.

$(2-i)^2 = 4 - 4i + i^2 = 3 - 4i$

$(2-i)^3 = (3-4i)(2-i) = 6 - 11i + 4i^2 = 2 - 11i$

$(2-i)^4 = (3-4i)^2 = 9 - 24i + 16i^2 = -7 - 24i$.

$2(-7-24i) - 3(2-11i) - 4(3-4i) + 2 - i + 30$

$-14 - 48i - 6 + 33i - 12 + 16i + 2 - i + 30$

$-14 - 6 - 12 + 2 + 30 - 48i + 33i + 16i - i = 0 + 0i$ $\therefore$ $2-i$ is a root.

(b) if $z = 2-i$ $\therefore$ $z = 2+i$

$(z-2+i)(z-2-i) \Rightarrow (z-2)^2 - i^2$

$\rightarrow z^2 - 4z + 4 - (-1) = z^2 - 4z + 5$

(c)
$$
\begin{array}{r}
2z^2 + 5z + 6 \\
z^2-4z+5 \,\overline{)\, 2z^4 - 3z^3 - 4z^2 + z + 30} \\
-\underline{2z^4 - 8z^3 + 10z^2} \\
5z^3 - 14z^2 + z \\
-\underline{5z^3 - 20z^2 + 25z} \\
6z^2 - 24z + 30 \\
-\underline{6z^2 - 24z + 30} \\
0
\end{array}
$$

$$\frac{-5 \pm \sqrt{25 - 4(6)(2)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{-23}}{4} = -\frac{5}{4} \pm \frac{\sqrt{23}i}{4}$$

$$z_3 = \frac{-5}{4} + \frac{\sqrt{23}i}{4}$$

$$z_4 = -\frac{5}{4} - \frac{\sqrt{23}i}{4}$$