1 A car of mass 960 kg is moving on a straight horizontal road. There is a constant force of magnitude 620 N resisting the motion of the car.

(a) Calculate the power developed by the engine of the car when it is moving at a constant speed of $25\,\text{m s}^{-1}$. [2]

Since, there is a constant $(R_f) \longleftarrow \boxed{\text{CAR}} \longrightarrow (D.F.)$

speed, $D.F. = R_f = 620\text{N}$

$\therefore \text{Power} = (D.F.) \times V$

$= 620 \times 25$

$= 15500\,\text{W}$

$\boxed{P = 15.5\,\text{kW}}$

(b) Given that the power is suddenly increased **by** 12 kW, find the instantaneous acceleration of the car. [2]

New power $= 15.5 + 12 = 27.5\,\text{kW}$

$= 27500\,\text{W}.$

Since the instantaneous acceleration is asked, speed will stay the same till that instant.
(constant)

New $D.F. = \dfrac{27500}{25} = 1100\text{N}$

$\therefore 1100 - 620 = m \times a$

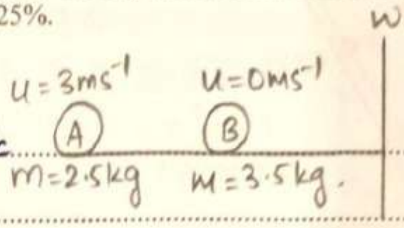
$480 = 960\,a$

$\boxed{\therefore a = 0.5\,\text{ms}^{-2}}$

2 Two particles, A and B, of masses 2.5 kg and 3.5 kg respectively are at rest on a straight smooth horizontal track. Particle B is situated 9 m from a vertical wall which is fixed at right angles to the track.

Particle A is projected directly towards B with a speed of $3\,\mathrm{m\,s^{-1}}$. In the subsequent motion, A collides and coalesces with B to form particle C. Particle C then collides directly with the wall and rebounds. The collision of C with the wall reduces the speed of C by 25%.

(a) Find the speed of C after it rebounds from the wall. [2]



$(2.5\times3) + (3.5\times0) = (2.5+3.5)V_c$

$7.5 = 6V_c$

$\boxed{V_c = 1.25\,ms^{-1}}$ → Velocity after coalition.

After rebound,

$V_c = 1.25\times0.75$

$V_c = \frac{15}{16} = \underline{\underline{0.9375\,ms^{-1}}}$

(b) Hence find the time from the instant at which A and B collide until C is once again a distance of 9 m from the wall. [2]

Time taken to reach the wall after A and B collides

$t_1 = \frac{9}{1.25} = \underline{\underline{7.2\,s}}$

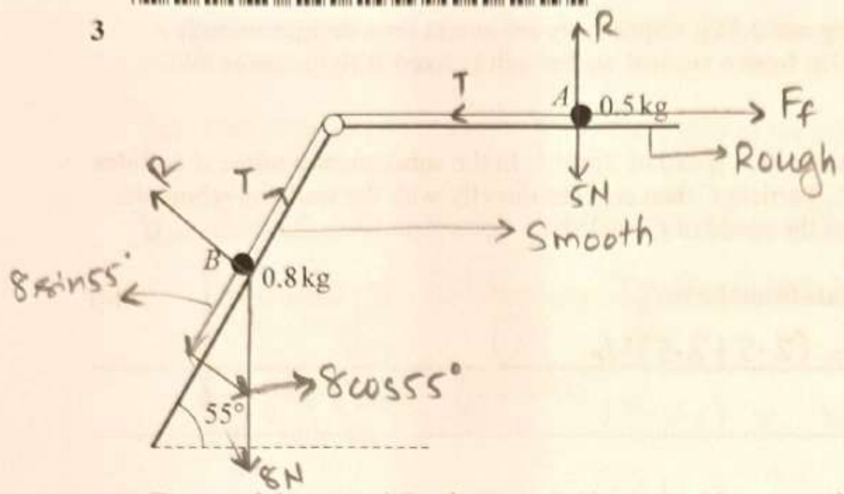
Time taken to reach again a distance of 9 m from the wall,

$t_2 = \frac{9}{0.9375} = 9.6\,s$

$\therefore$ Total time $= 7.2 + 9.6$

$= \underline{\underline{16.8\,s}}$

3



Two particles, A and B, of masses 0.5 kg and 0.8 kg respectively are attached to the ends of a light inextensible string. The string passes over a small smooth pulley fixed at the end of a rough horizontal plane and to the top of a smooth inclined plane. Particle A is held on the horizontal plane, while B lies on the inclined plane, which makes an angle of 55° with the horizontal. The string is in the same vertical plane as a line of greatest slope of the inclined plane (see diagram).

Particle A is released from rest with the string taut. Particle B moves 0.3 m down the inclined plane in 0.4 s.

(a) Find the tension in the string. [4]

Since, the system is released from rest,

$u = 0$, $s = 0.3\text{m}$, $t = 0.4\text{s}$

$\therefore s = ut + \frac{1}{2}at^2 \Rightarrow 0.3 = 0 + \frac{1}{2}(a)(0.4)^2$

$\therefore a = \frac{0.6}{(0.4)^2} = \underline{\underline{3.75\text{ms}^{-2}}}$

Using N2LM on particle B,

$8\sin 55° - T = (0.8)(3.75)$

$\therefore T = 8\sin 55 - (0.8 \times 3.75)$

$T = 3.553216 \ldots$

$\boxed{T \simeq 3.55\text{N}}$

(b) Find the coefficient of friction between A and the horizontal plane. [3]

$R = 5N$; $F_f = \mu(5) = 5\mu$

Using N2LM on particle A,

$T - F_f = m \times a$

$3.553216.... - 5\mu = 0.5 \times 3.75$

$\therefore \mu = \frac{3.553216.... - (0.5 \times 3.75)}{5}$

$\therefore \mu = 0.33564....$

$\boxed{\therefore \mu = 0.336}$

(c) After B has been moving for 0.4 s, the string suddenly breaks. Given that A subsequently comes to rest on the horizontal plane, find the work done by the frictional force in bringing A to rest. You may assume that A does not reach the pulley. [3]

Since the string suddenly breaks, hence, $T = 0$.

Hence, $-F_f = m \times a \Rightarrow -5 \times 0.336 = 0.5 \times a$

$\therefore a = -3.3564... ms^{-2}$

In $t = 0.4s$, $V^2 = u^2 + 2as = 2(3.75)(0.3) =$

$\therefore V = 1.5 ms^{-1}$

After 0.4s, the string breaks. ($u = 1.5ms^{-1}$, $V = 0$,

$0 = (1.5)^2 + 2(-3.356...)(s)$ $a = -3.356....$, $s = ?$

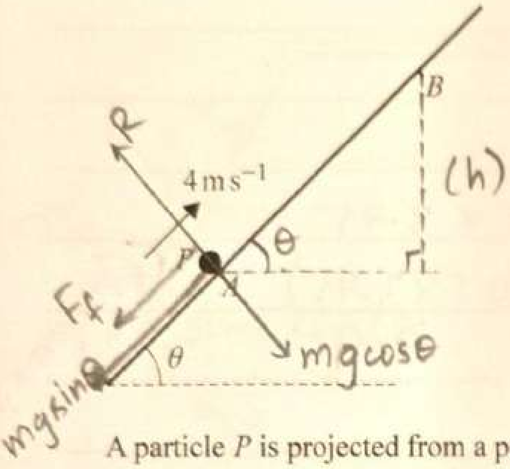
$S = 0.335m$

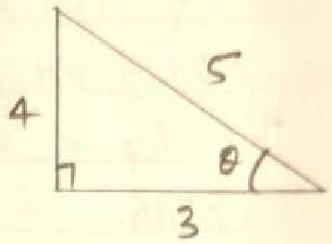
$\therefore$ Work done by $F_f = (5 \times 0.33564....) \times 0.335$

$= 0.56256...$

$\boxed{(W.D)_{F_f} \approx 0.563 J}$

4





$\sin\theta = \frac{4}{5}$; $\cos\theta = \frac{3}{5}$

A particle P is projected from a point A with speed $4\,\text{m s}^{-1}$ up a line of greatest slope of a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

P comes to instantaneous rest at a point B on the plane (see diagram).

The coefficient of friction between P and the plane is $\frac{1}{3}$. (μ)

(a) Using an energy method, show that the distance AB is $0.8\,\text{m}$. [4]

Change in K.E. $= \frac{1}{2}m(0^2 - 4^2) = -8m$

(Loss)

Change in G.P.E $= m \times 10 \times (AB\sin\theta)$ $\quad \sin\theta = \frac{h}{AB}$

$= m \times 10 \times \frac{4}{5} \times (AB)$

$= (AB)\cdot(8m)$

Work done against friction = ~~$F_f \times (AB)$~~ $(F_f + m\sin\theta)\cdot AB$

= ~~$(\mu R) \times (AB)$~~

$= \left(\frac{m\cos\theta}{3} + m\sin\theta\right)AB$ ~~$= \frac{1}{3} \times m\cos\theta \times AB$~~

$= (0.2m + 0.8m)AB$ ~~$= \frac{1}{3} \times m \times \frac{3}{5} \times AB = (AB)\cdot(0.2m)$~~

Applying energy method,

$-8m + (AB)\cdot(8m) = -(AB) \times$ ~~$(0.2m)$~~ $(1m)$

~~$AB[8m + 0.2m] = 8m$~~

~~$AB = 8m$~~ $\quad -8 + 8AB = -AB$

$9AB = 8$ $\quad \boxed{\therefore AB = 0.8\,m}$

(b) After coming to instantaneous rest at B, the particle slides down the plane.

Find the total time from the instant at which P is projected from A until it returns to A. [5]

Time taken to reach ~~A~~ from A to B:- (t_1)

$u = 4\,ms^{-1}$, $s = 0.8\,m$, $v = 0$, $t = ?$

$s = \frac{1}{2}(u+v)t_1 \Rightarrow$ $0.8 = \frac{1}{2}(4+0)\cdot t_1$

$\boxed{\therefore t_1 = 0.4\,s}$

Time taken to travel from B to A :- (t_2)

$u_{down} = 0\,ms^{-1}$, $s = 0.8\,m$, $a_{downward} = ?$, $t_2 = ?$

$\Sigma F_{net} = ma$.

$mg\sin\theta - F_f = ma$

$m \times 10 \times \frac{4}{5} - \frac{1}{3} \times m \times 10 \times \frac{3}{5} = m \times a_{down}$

$8m - 2m = m \times a_{down}$

$\therefore a_{down} = 6\,ms^{-2}$

$s = u_{down}t + \frac{1}{2}a_{down}t_2^2$

$0.8 = 0 + \frac{1}{2}(6)t_2^2 \Rightarrow 3t_2^2 = 0.8$

$\Rightarrow t_2 = 0.51639\ldots\, s$

$\therefore$ Total time taken $= t_1 + t_2 = 0.91639\ldots \simeq \boxed{0.916\,s}$

(a) Continuation:-

Work done against friction

$= F_f \times AB = \mu R \times AB$

$= \frac{1}{3} \times mg\cos\theta \times AB$

$= \frac{1}{3} \times m \times 10 \times \frac{3}{5} \times AB$

$= (2m)\cdot(AB)$

Using energy principle,

$-8m + 8m\cdot(AB) = -(2m)AB$

$AB \cdot 10m = 8m$

$\therefore AB = \frac{8m}{10m}$

$\boxed{\therefore AB = 0.8\,m}$

5 The points A and B are at the same vertical height h m above horizontal ground. A particle P is released from rest from A. One second later, a particle Q is projected vertically downwards from B with speed $18\,\text{m}\,\text{s}^{-1}$.

Given that P and Q reach the ground at the same time, find the value of h. [6]

Let particle P is released from rest from A @ time t.

Hence, particle Q is projected after (t-1) s.

* FOR PARTICLE P:- (t s)

$h_P = ut + \frac{1}{2}at^2$

$h_{AP} = 0 + \frac{1}{2}(10)t^2$

$h_{AP} = 5t^2$ ——— ①

* FOR PARTICLE Q:- (t-1) s

$h_Q = 18(t-1) + \frac{1}{2}(10)(t-1)^2$

$h_Q = 18t - 18 + 5(t^2 - 2t + 1)$

$h_Q = 18t - 18 + 5t^2 - 10t + 5$

$h_Q = 5t^2 + 8t - 13$

Since same vertical height,

$\therefore h_P = h_Q$

$5t^2 = 5t^2 + 8t - 13$

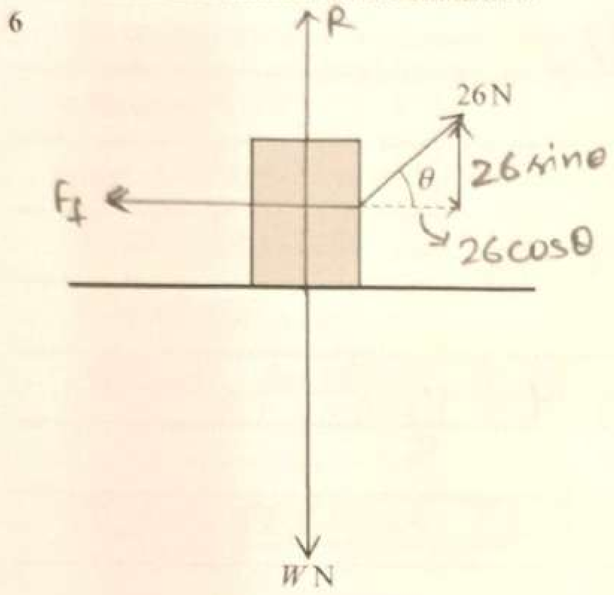
$8t = 13 \Rightarrow t = \frac{13}{8}$ s

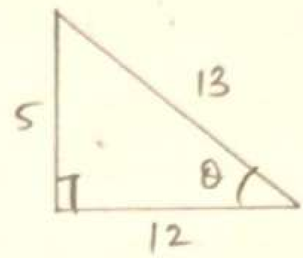
$\therefore h = 5\left(\frac{13}{8}\right)^2$

$h = 13.203125$m

$\boxed{h \approx 13.20\text{m}}$

6





$\cos\theta = \frac{12}{13}$;

$\tan\theta = \frac{5}{12}$

A crate of weight W N is at rest on rough horizontal ground. The crate is pulled at a constant acceleration of $1.5\,\mathrm{m\,s^{-2}}$ along the ground in a straight line by a light rope. The rope is inclined at an angle of θ to the horizontal, where $\theta = \sin^{-1}\frac{5}{13}$. The tension in the rope is 26 N (see diagram). The coefficient of friction between the crate and the ground is 0.25.

(a) Find the value of W. [5]

Using N2LM, $\Sigma F_{net} = ma$.

(*) Resolving Vertically:-

$R + 26\sin\theta - W = 0$

$R = W - 26\sin\theta \quad \therefore R = W - 26\left(\frac{5}{13}\right)$

$\therefore R = W - 10$ ———— (1)

(*) Resolving horizontally:-

$26\cos\theta - F_f = ma$

$26 \times \frac{12}{13} - (\mu \times R) = \left(\frac{W}{10}\right) \times 1.5$

$24 - (0.25 \times (W - 10)) = 0.15W$

$24 - 0.25W + 2.5 = 0.15W$

$26.5 = 0.40W$

$\therefore \boxed{W = \frac{265}{4} = 66.25\,N}$

(b) When the crate reaches a point A, the rope is removed. The speed of the crate at A is $8\,m\,s^{-1}$.

The crate comes to rest at a point B.

Find the distance AB. [3]

Between A and B, there is no 26N force anymore.

$\therefore -F_f = m \times a$. Also, $R = W = 66.25\,N$

$\therefore -0.25 \times 66.25 = \left(\frac{W}{10}\right) \times (a)$

$-\frac{265}{16} = 6.625 \times a$

$a = -2.5\,ms^{-2}$

$V^2 = u^2 + 2as$

$0 = (8)^2 + 2(-2.5)(s)$

$5s = 64$

$s = \frac{64}{5}$

$\boxed{\therefore s = 12.8\,m}$

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7 A particle P moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P at time t s, where $t \geqslant 0$, is given by

$$v = k_1(4t+11)^{\frac{1}{2}} - \tfrac{1}{4}(2t+1)^2 + k_2,$$

where k_1 and k_2 are constants. When $t = 1.25$ the deceleration of P is $0.5\,\mathrm{m\,s^{-2}}$.

(a) Find the value of k_1. [4]

$a = \frac{dv}{dt} = k_1 \cdot \frac{4}{2}(4t+11)^{-\frac{1}{2}} - \frac{2}{4}\cdot 2(2t+1)^{1} + 0$

$\therefore a = \frac{2k_1}{\cancel{}}(4t+11)^{-1/2} - \frac{1}{\cancel{}}(2t+1)$

When $t = 1.25\,s$, $a = -0.5\,\mathrm{ms^{-2}}$

$-0.5 = \frac{2k_1}{\cancel{}}\left[4\times1.25+11\right]^{-\frac{1}{2}} - \frac{1}{\cancel{}}\left[2\times1.25+1\right]$

$\cancel{-1 = k_1\left(\frac{1}{4}\right) - \frac{7}{2}}$

$\cancel{-4 = k - 14}$

$\therefore \cancel{k =}$

$-0.5 = 2k_1\left(\frac{1}{4}\right) - \frac{7}{2}$

$3 = \frac{2k_1}{4}$

$\boxed{\therefore k_1 = 6}$

(b) Given that P is only at instantaneous rest when $t = 3.5$, find the distance travelled by P in the interval $0 \leqslant t \leqslant 1$. [5]

$V = 6(4t+11)^{\frac{1}{2}} - \frac{1}{4}(2t+1)^2 + k_2$

when $t = 3.5$, $V = 0$.

$\therefore 0 = 6(4 \times 3.5 + 11)^{1/2} - \frac{1}{4}(2 \times 3.5 + 1)^2 + k_2$

$0 = 30 - 16 + k_2$

$k_2 = -14$

$\therefore V = 6(4t+11)^{\frac{1}{2}} - \frac{1}{4}(2t+1)^2 - 14$

$S = \int_0^1 V \cdot dt = \int_0^1 \left[6(4t+11)^{1/2} - \frac{1}{4}(2t+1)^2 - 14\right] \cdot dt$

$= \left[\frac{6(4t+11)^{3/2}}{4 \times \frac{3}{2}} - \frac{(2t+1)^3}{4 \times 3 \times 2} - 14t\right]_0^1$

$= \left[(4t+11)^{3/2} - \frac{(2t+1)^3}{24} - 14t\right]_0^1$

$= \left[(4 \times 1 + 11)^{3/2} - \frac{(2 \times 1 + 1)^3}{24} - (14 \times 1)\right] - \left[(4 \times 0 + 11)^{3/2} - \frac{(2 \times 0 + 1)^3}{24} - (14 \times 0)\right]$

$\boxed{S = 6.53\,m}$