A-Level Mathematics Paper 52 (Statistics 1)

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Contents

Question 1

A club has 15 members made up of 6 men and 9 women. Every Friday a team of 5 members is selected at random to represent the club in a competition.

(a) Find the probability that next Friday the team will consist of more men than women. [3]

/ Solution

Let the team consist of M men and W women. For more men than women, we need $(M, W) = (3, 2), (4, 1), \text{ or } (5, 0)$.

$$P = \frac{C_6^3 \times C_9^2 + C_6^4 \times C_9^1 + C_6^5 \times C_9^0}{C_{15}^5}$$

$$P = \frac{20 \times 36 + 15 \times 9 + 6 \times 1}{3003}$$

$$P = \frac{720 + 135 + 6}{3003} = \frac{861}{3003} \approx 0.287$$

(b) Another club has 13 members. In how many ways can the group of 13 members be divided into a group of 6, a group of 4 and a group of 3? [2]

/ Solution

This is a division into distinct group sizes.

$$\text{Total ways} = C_{13}^6 \times C_7^4 \times C_3^3$$

$$\text{Total} = 1716 \times 35 \times 1 = 60060$$

Question 2

Ling has three coins. Each coin is biased so that the probability of obtaining a head is $\frac{1}{3}$. Ling throws all three coins at the same time. The random variable X denotes the number of heads that Ling obtains.

(a) Draw the probability distribution table for X. [3]

/ Solution

$X \sim B(3, \frac{1}{3})$. We calculate each probability:

$$P(X=0) = C_3^0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$P(X=1) = C_3^1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^2 = \frac{12}{27} = \frac{4}{9}$$

$$P(X=2) = C_3^2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 = \frac{6}{27} = \frac{2}{9}$$

$$P(X=3) = C_3^3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^0 = \frac{1}{27}$$

x	0	1	2	3
$P(X=x)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

(b) Ling throws all three coins at the same time repeatedly. Find the probability that he obtains at least two heads for the first time on his 7th throw. [2]

/ Solution

Let S be the event of obtaining at least two heads in a single throw.

$$P(S) = P(X \geq 2) = P(X=2) + P(X=3) = \frac{6}{27} + \frac{1}{27} = \frac{7}{27}$$

For the first success to occur on the 7th throw, we need 6 failures followed by 1 success.

$$P(1^{\text{st}} \text{ success on } 7^{\text{th}}) = (1 - P(S))^6 \times P(S) = \left(\frac{20}{27}\right)^6 \times \left(\frac{7}{27}\right) \approx 0.0428$$

(c) Find the probability that he obtains at least two heads for the second time on his 7th throw. [3]

/ Solution

For the second success to occur on the 7th throw, there must be exactly 1 success in the first 6 throws, and a success on the 7th throw.

$$P(2^{\text{nd}} \text{ success on } 7^{\text{th}}) = \left[C_6^1 \left(\frac{7}{27}\right)^1 \left(\frac{20}{27}\right)^5\right] \times \frac{7}{27}$$

$$= 6 \times \left(\frac{7}{27}\right)^2 \times \left(\frac{20}{27}\right)^5 \approx 0.0894$$

Question 3

(a) Find the number of different 3-digit numbers greater than 200 from the digits $\{1, 2, 3, 4, 5, 6, 7\}$ if digits may be repeated and the number is divisible by 5. [3]

/ **Solution**

The number is divisible by 5, so the last digit must be 5 (since 0 is not available).

- Last digit: 1 choice (5).
- First digit: Since the number > 200, it can be 2, 3, 4, 5, 6, 7. Thus, 6 choices.
- Middle digit: Can be any of the 7 digits (repetition allowed). Thus, 7 choices.

Total numbers $= 6 \times 7 \times 1 = 42$.

(b) Find the number of different 3-digit numbers greater than 200 that can be made from the digits $\{1, 2, 3, 4, 5, 6, 7\}$ if no digit is repeated and the number is even. [3]

/ **Solution**

The number is even, so the last digit must be 2, 4, or 6. We consider each case to avoid overlap with the > 200 condition for the first digit.

- **Case 1: Last digit is 2.** First digit can be 3, 4, 5, 6, 7 (5 choices). Middle digit has $7 - 2 = 5$ choices. Total $= 1 \times 5 \times 5 = 25$.
- **Case 2: Last digit is 4.** First digit can be 2, 3, 5, 6, 7 (5 choices). Middle digit has 5 choices. Total $= 1 \times 5 \times 5 = 25$.
- **Case 3: Last digit is 6.** First digit can be 2, 3, 4, 5, 7 (5 choices). Middle digit has 5 choices. Total $= 1 \times 5 \times 5 = 25$.

Total numbers $= 25 + 25 + 25 = 75$.

Question 4

Every day Haz attempts to complete a particular type of puzzle in less than 10 minutes. On any attempt, the probability that she will succeed is 0.6.

(a) Find the probability that on 9 randomly chosen days, Haz completes the puzzle in less than 10 minutes on at least 7 of these 9 days. [3]

/ **Solution**

Let X be the number of successful days. $X \sim B(9, 0.6)$.

$$P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9)$$

$$P(X = 7) = C_9^7 (0.6)^7 (0.4)^2 \approx 0.16128$$
$$P(X = 8) = C_9^8 (0.6)^8 (0.4)^1 \approx 0.06047$$
$$P(X = 9) = C_9^9 (0.6)^9 (0.4)^0 \approx 0.01008$$
$$P(X \geq 7) = 0.16128 + 0.06047 + 0.01008 \approx 0.232 \text{ (to 3 s.f.)}$$

(b) 120 days are chosen at random. Use an approximation to find the probability that Haz completes the puzzle in less than 10 minutes on fewer than 66 of these 120 days. [5]

Solution

Let X be the number of successful days out of 120. $X \sim B(120, 0.6)$. Since n is large, we approximate using Normal Distribution $N(\mu, \sigma^2)$.

$$\mu = np = 120 \times 0.6 = 72$$

$$\sigma^2 = np(1-p) = 72 \times 0.4 = 28.8$$

We want to find $P(X < 66)$. Applying continuity correction, we calculate $P(X < 65.5)$:

$$Z = \frac{65.5 - 72}{\sqrt{28.8}} = \frac{-6.5}{5.3666} \approx -1.211$$

$$P(Z < -1.211) = 1 - \Phi(1.211) = 1 - 0.8871 = 0.113$$

Question 5

A box contains 3 red pens, 5 blue pens and 2 green pens. Lui chooses one pen at random from the box and keeps it. A green pen is added to the box. Mira now chooses one pen at random from the box and keeps it.

(a) Find the probability that Lui and Mira both have a blue pen. [1]

Solution

Lui picks Blue: $P(L_B) = \frac{5}{10}$. After Lui picks a blue pen, there are 4 blue pens left. A green pen is added, making the total 10 again. Mira picks Blue given Lui picked Blue: $P(M_B|L_B) = \frac{4}{10}$.

$$P(\text{Both Blue}) = \frac{5}{10} \times \frac{4}{10} = \frac{20}{100} = 0.2$$

(b) Find the probability that Mira's pen is red. [3]

Solution

Mira picking red (M_R) depends on Lui's pick. We sum the probabilities across all of Lui's possible choices (Red, Blue, Green).

$$P(M_R) = P(L_R \cap M_R) + P(L_B \cap M_R) + P(L_G \cap M_R)$$

- If Lui picks Red (3/10): 2 Red left, total becomes 10 (after green added). $M_R = 2/10$. Product $= \frac{3}{10} \times \frac{2}{10} = \frac{6}{100}$.
- If Lui picks Blue (5/10): 3 Red left, total 10. $M_R = 3/10$. Product $= \frac{5}{10} \times \frac{3}{10} = \frac{15}{100}$.
- If Lui picks Green (2/10): 3 Red left, total 10. $M_R = 3/10$. Product $= \frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$.

$$P(M_R) = \frac{6}{100} + \frac{15}{100} + \frac{6}{100} = \frac{27}{100} = 0.27$$

(c) Find the probability that Lui's pen is red given that Mira's pen is blue or green. [3]

/ Solution

We want $P(L_R|M_{B\cup G})$. Notice that Mira picking Blue or Green is the complement of Mira picking Red.

$$P(M_{B\cup G}) = 1 - P(M_R) = 1 - 0.27 = 0.73$$

Now, find the intersection $P(L_R \cap M_{B\cup G})$. If Lui picks Red ($P = 3/10$), the new box has 2 Red, 5 Blue, and 3 Green. Mira picking Blue or Green has probability $\frac{5+3}{10} = \frac{8}{10}$.

$$P(L_R \cap M_{B\cup G}) = \frac{3}{10} \times \frac{8}{10} = \frac{24}{100} = 0.24$$

Using conditional probability formula:

$$P(L_R|M_{B\cup G}) = \frac{P(L_R \cap M_{B\cup G})}{P(M_{B\cup G})} = \frac{0.24}{0.73} = \frac{24}{73} \approx 0.329$$

Question 6

The heights, in cm, of the 15 rugby players in each of two teams, the Hippos and the Zebras, are shown in the table.

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Hippos on the left-hand side. [3]

/ Solution

Hippos	Stem	Zebras
8, 5	**16**	7
8, 6, 2, 2	**17**	1, 2, 5, 7, 9
8, 8, 5, 4, 1	**18**	2, 3, 4, 6, 6, 9
5, 2, 1, 0	**19**	1, 2, 4

Key: 5 | **16** | 7 represents 165 cm for Hippos and 167 cm for Zebras.

(b) Find the interquartile range of the heights of the players in the Hippos team. [2]

/ Solution

There are $n = 15$ players in Hippos. Data ordered: 165, 168, 172, 172, 176, 178, 181, **184**, 185, 188, 188, 190, 191, 192, 195.

- Median (Q_2) is the 8th value = 184 cm.
- Lower Quartile (Q_1) is the 4th value = 172 cm.
- Upper Quartile (Q_3) is the 12th value = 190 cm.

$$\text{IQR} = Q_3 - Q_1 = 190 - 172 = 18 \text{ cm}$$

(c) The player in the Hippos with height 184 cm is injured. A new player replaces him, and this causes the mean height of the 15 players in the team to increase by 0.5 cm. Find the height of the new player. [2]

/ **Solution**

Let the new player's height be x_{new}. The change in the sum of heights is $\Delta\Sigma x = 15 \times 0.5 = 7.5$ cm. This change corresponds to the difference between the new player and the injured player:

$$x_{new} - 184 = 7.5$$

$$x_{new} = 191.5 \text{ cm}$$

Question 7

The lengths of the leaves of a certain type of tree are normally distributed with mean μ cm and standard deviation σ cm. It is known that 20% of the leaves have a length less than 3.60 cm and 70% of the leaves have a length greater than 5.40 cm.

(a) Find the value of μ and the value of σ. [5]

/ **Solution**

From the given probabilities: 1) $P(X < 3.60) = 0.20 \implies Z_1 = \Phi^{-1}(0.2) = -0.842$
2) $P(X > 5.40) = 0.70 \implies P(X < 5.40) = 0.30 \implies Z_2 = \Phi^{-1}(0.3) = -0.524$
Using the standardizing formula $Z = \frac{X-\mu}{\sigma}$:

$$-0.842 = \frac{3.60 - \mu}{\sigma} \implies \mu - 0.842\sigma = 3.60 \quad (1)$$

$$-0.524 = \frac{5.40 - \mu}{\sigma} \implies \mu - 0.524\sigma = 5.40 \quad (2)$$

Subtracting (1) from (2):

$$0.318\sigma = 1.80 \implies \sigma \approx 5.66$$

Substitute σ back into (1):

$$\mu = 3.60 + 0.842(5.66) \approx 3.60 + 4.765 = 8.365 \approx 8.37$$

Hence, $\mu \approx 8.37$ and $\sigma \approx 5.66$.

(b) The lengths of 600 randomly chosen leaves of a different type of tree are normally distributed with mean 4.63 cm and standard deviation 0.55 cm. How many of these 600 leaves would you expect to have a length within 0.6 cm of the mean length? [4]

/ **Solution**

Let $Y \sim N(4.63, 0.55^2)$. We want to find $P(\mu - 0.6 < Y < \mu + 0.6)$, which is $P(4.03 < Y < 5.23)$.

$$Z_1 = \frac{4.03 - 4.63}{0.55} = \frac{-0.6}{0.55} \approx -1.091$$

$$Z_2 = \frac{5.23 - 4.63}{0.55} = \frac{0.6}{0.55} \approx 1.091$$

$$P(-1.091 < Z < 1.091) = 2\Phi(1.091) - 1$$

Using Normal tables, $\Phi(1.091) \approx 0.8623$

$$P = 2(0.8623) - 1 = 0.7246$$

Expected number of leaves $= 600 \times 0.7246 = 434.76 \approx 435$ leaves.