1 Ten random values of a variable X are given below.

8.1	8.3	9.5	10.1	7.9	8.7	9.1	8.8	9.4	9.0

Calculate unbiased estimates of the population mean and variance of X.

2 Each week, Birgitte takes three train journeys. The times, in minutes, for these journeys have the independent distributions $A \sim \mathrm{N}(28, 15)$, $B \sim \mathrm{N}(23, 8)$ and $C \sim \mathrm{N}(99, 24)$.

For a randomly chosen week, find $\mathrm{P}(C > 2(A + B))$. [5]

3 The number of vehicles, X, passing a certain junction between 1.00 pm and 1.01 pm is modelled by a Poisson distribution. In the past, the population mean of X was 2.4. After a change is made to the junction, the value of X is noted on a randomly chosen day. A hypothesis test at the 5% significance level is carried out to determine whether the population mean of X has increased.

(a) State suitable null and alternative hypotheses for the test. [1]

....

....

(b) Find the smallest value of X that would result in rejecting the null hypothesis. [4]

(c) State the value of P(Type I error).

4 The random variables D and E have the independent distributions $D \sim \text{Po}(1.7)$ and $E \sim \text{Po}(2.1)$.

(a) Find $P(D+E < 4)$. [3]

(b) Given that $D + E < 4$, find $P(D = 1)$.

(c) The random variable T is the total of 100 randomly chosen values of D.

Use the normal approximation to the Poisson distribution to find $\mathrm{P}(T > 190)$.

5 Eric throws darts at a target. The probability that he will hit the centre of the target on one throw is p.

(a) Eric takes 100 throws and hits the centre x times. He uses this result to find an approximate 95% confidence interval for p. The width of this interval is 0.165 correct to 3 significant figures.

Find the two possible values of x. [4]

Eric takes some more throws. He uses his results to calculate two approximate α % confidence intervals for p (where $\alpha > 90$).

It is given that the probability that exactly one of these two intervals contains the true value of p is 0.1128.

(b) Find the value of α. [3]

6 The length of time, T minutes, that customers spend in a certain shop has the distribution $N(\mu, 9.2)$. Last year the value of μ was 12.4. The manager of the shop wants to test whether the value of μ has decreased this year. She chooses a random sample of 75 customers and finds that the sample mean, $\bar{T}$, for these customers is 11.9.

(a) Test at the 5% significance level whether the value of μ is less than 12.4. [5]

(b) Explain whether it was necessary to use the Central Limit Theorem in your answer to **6(a)**.

....

(c) Later the manager chooses another large random sample of n customers. She finds that, for these customers, $T = 11.8$. She carries out a similar test at the 5% significance level, and the conclusion is that there is evidence that the value of μ is less than 12.4.

Find the range of possible values of n. [3]

7 A random variable X has the probability density function

$$f(x) = \begin{cases} a - x & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Show that $E(X) = k\sqrt{2}$, where k is a constant to be found.

[6]

(b) *Find the exact value of the median of X.* [4]