2025

AP® Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 4

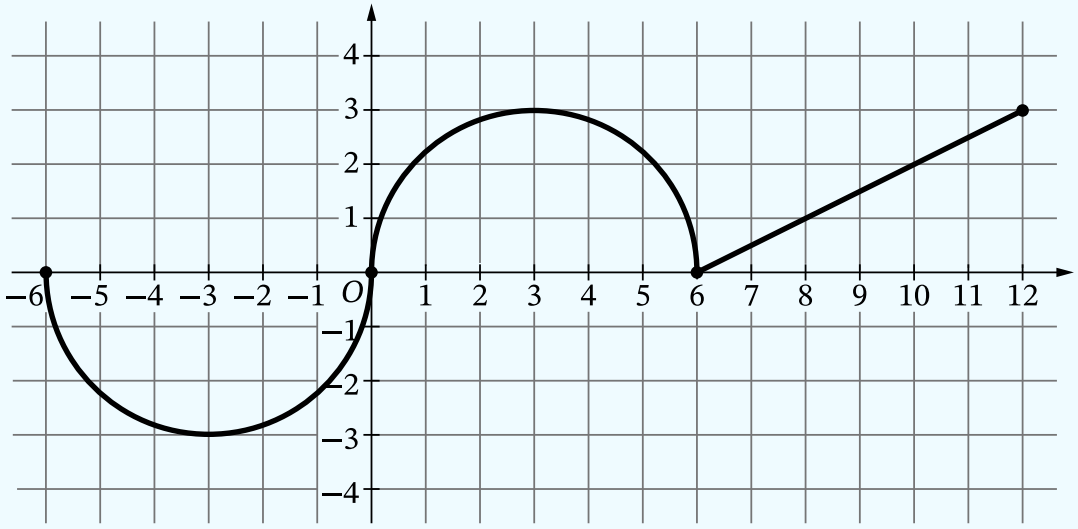
- ☑ **Scoring Guidelines**
- ☑ **Student Samples**
- ☑ **Scoring Commentary**

Part A (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.

	Model Solution	Scoring
A	Find $g'(8)$. Give a reason for your answer.	
	$g'(x) = f(x)$	Considers $g'(x) = f(x)$ **Point 1 (P1)**
	$g'(8) = f(8) = 1$	Answer **Point 2 (P2)**
	Scoring Notes for Part A	

- **P1** is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A.
- A response of $g'(8) = f(8) = 1$ earns both **P1** and **P2**.
- A response that does not earn **P1** can earn **P2** with an implied application of the Fundamental Theorem of Calculus (e.g., $g'(8) = 1$ or $f(8) = 1$).
- A response of $g'(8) = f(8) - f(6) = 1$ earns **P2** but not **P1**.

B Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

The graph of g has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa. The graph of g has points of inflection at $x = -3$ and $x = 6$ because f changes from decreasing to increasing there. The graph of g also has a point of inflection at $x = 3$ because f changes from increasing to decreasing there.	Answer	**Point 3 (P3)**
	Reason	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of x in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of f.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f changes from increasing to decreasing or decreasing to increasing there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of f changes sign there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f attains relative extrema there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because g changes concavity there" earns **P3** but not **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there" earns **P3** but not **P4**.
  - A response that relies upon an ambiguous term such as "the function" or "the graph" does not earn **P4**.
- **Special case:** A response with two of the three correct x-values with correct reasoning and no other/additional values of x declared to be points of inflection earns **P4** but not **P3**.

C Find $g(12)$ and $g(0)$. Label your answers.

$g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2} \cdot 6 \cdot 3 = 9$	$g(12)$	**Point 5 (P5)**
$g(0) = \int_6^0 f(x)\,dx = -\int_0^6 f(x)\,dx = -\frac{\pi}{2}3^2 = -\frac{9\pi}{2}$	$g(0)$	**Point 6 (P6)**

Scoring Notes for Part C

- Unlabeled values do not earn either **P5** or **P6**.
- **P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- **P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.

  Note: Incorrect communication between the label "$g(0)$" and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^6 f(x)\,dx = -\frac{9\pi}{2}$ earns **P6**.

D Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

For $-6 \le x \le 12$, g attains a minimum either when $g'(x) = f(x) = 0$ or at an endpoint. $g'(x) = f(x) = 0$	Considers $g'(x) = 0$ **Point 7 (P7)**
$\Rightarrow \quad x = 0, \; x = 6$ $\begin{array}{c\|c} x & g(x) \\ \hline -6 & 0 \\ 0 & -\frac{9\pi}{2} \\ 6 & 0 \\ 12 & 9 \end{array}$	Justification **Point 8 (P8)**
Therefore, on the closed interval $-6 \le x \le 12$, g attains an absolute minimum value at $x = 0$.	Answer **Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.

  A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase "critical points of g" also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other x-values).
- Alternate justification and answer:

  Because $g'(x) \le 0$ (or $f(x) \le 0$) for $-6 \le x < 0$ and $g'(x) \ge 0$ (or $f(x) \ge 0$) for $0 < x \le 12$, the absolute minimum of g occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$\frac{d}{dx} g(x) = \frac{d}{dx} \int_0^x f(t)\,dt$

$g'(x) = f(x)$

$g'(8) = f(8) = \boxed{1}$

fundamental theorem of calculus states that $\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$

PART B

g has a pt of inflection at $x = -3$ because f (or g') changes from decreasing to increasing

g has a pt of inflection at $x = 3$ because f (or g') changes from increasing to decreasing

g has a pt of inflection $x = 6$ because f (or g') changes from decreasing to increasing

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS C and D on this page.

PART C

$$g(12) = \int_6^{12} f(t)\,dt$$

$$g(12) = \frac{(6)(3)}{2}$$

$$\boxed{g(12) = 9}$$

$$g(0) = \int_6^0 f(t)\,dt$$

$$g(0) = -\int_0^6 f(t)\,dt$$

$$g(0) = -\left(\tfrac{1}{2}\pi(3)^2\right)$$

$$g(0) = -\left(\tfrac{9}{2}\pi\right)$$

$$\boxed{g(0) = -\tfrac{9}{2}\pi}$$

PART D

g is continuous on $[-6, 12]$

$\therefore$ EVT applies

$$g'(x) = f(x)$$

$$0 = f(x)$$

$$x = 0 \quad x = 6$$

x	$g(x)$
-6	0
0	$-\frac{9}{2}\pi$
6	0
12	9

$$g(0) < g(-6) = g(6) < g(12)$$

$\therefore$ g attains an absolute minimum value at $x = 0$

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$g'(x) = f(x)$

$g'(8) = f(8)$

$g'(8) = 1$

$f(8)$ appears to be 1 when looking at the graph

PART B

when $x = 0$ there appears to be a point of inflection as f goes from concave up to concave down.

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS C and D on this page.

PART C

$g(12) = 9$

$g(0) = -\frac{9}{2}\pi$

PART D

$g'(x) = f(x) = 0$

$x = 0, 6$

minimum when $x = 0$
knowing that $g'(x) = f(x)$
using the FTC, 0 is
a minimum as $f(0) = 0$
meaning the slope of g
0 and f goes from negative
to positive at that point making
it a minimum.

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$g(x) = \int_6^x f(t)\,dt$

$g'(x) = f(x)$

$g'(8) = f(8)$

$g'(8) = 1$

$g'(8) = 1$ because $g'(x) = f(x)$, when $x = 8$, $f(8) = 1$ and because the derivative of $g(x)$ equals $f(x)$, $g'(8)$ will also equal 1.

PART B $g''(x) = f'(x)$

POI: $x = 0, 6$

Because $g''(x) = f'(x)$, when the slope of $f(x)$ changes signs, $g(x)$ will have a point of inflection.

Q4 ☒ NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS C and D on this page.

PART C

$g(x) = \int_6^x f(t)\,dt$

$g(12) = \int_6^{12} f(t)\,dt$

$\boxed{g(12) = 18}$

$g(0) = \int_6^0 f(t)\,dt = -\int_0^6 f(t)\,dt$

$g(0) = -\frac{1}{2}\pi(3)^2$

$\boxed{g(0) = -\frac{9\pi}{2}}$

PART D

$g'(x) = f(x)$

$g(6) = 0$

$g(-6) = 0$

$g(12) = 18$

g attains an absolute maximum at $x = 12$ because $g(x) = \int_6^x f(t)\,dt$ and at $x = 6$, the integral equals 12, so $g(6) = 12$.

Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on AP Central.

Sample: 4A
Score: 9 (1-1-1-1-1-1-1-1-1)

The response earned 9 points: 2 points in part A, 2 points in part B, 2 points in part C, and 3 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 2. The response earned **P2** with the statement $g'(8) = f(8) = 1$ in line 3. This statement alone would have earned both **P1** and **P2**. Note: The comment presented on the right regarding the Fundamental Theorem of Calculus was not considered in scoring **P1** or **P2**.

In part B the response earned **P3** with the presentation of the correct answer $x = -3$, $x = 3$, and $x = 6$, with no additional values presented. The response earned **P4** with the correct reasoning that "f (or g') changes from decreasing to increasing" in the first and third sentences and "f (or g') changes from increasing to decreasing" in the second sentence.

In part C the response earned **P5** with the boxed statement $g(12) = 9$ in line 3 on the left. The numerical expression $g(12) = \frac{(6)(3)}{2}$ in line 2 on the left would have earned the point with no simplification. The response earned **P6** with the boxed statement $g(0) = -\frac{9}{2}\pi$ in line 5 on the right. The numerical expression $g(0) = -\left(\frac{1}{2}\pi(3)^2\right)$ in line 3 on the right would have earned the point with no simplification.

In part D the response earned **P7** with the statement $0 = f(x)$ in line 4 on the left. The response earned **P8** with the presentation of the correct table on the right. The response earned **P9** for the boxed statement that "g attains an absolute minimum value at $x = 0$" on the right.

Sample: 4B
Score: 6 (1-1-0-0-1-1-1-0-1)

The response earned 6 points: 2 points in part A, 0 points in part B, 2 points in part C, and 2 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 1. The response also would have earned this point with the equation $g'(8) = f(8)$ in line 2. The response earned **P2** with the equation $g'(8) = 1$ in line 3.

In part B the response did not earn **P3** because the answer $x = 0$ in line 1 is incorrect. The response did not earn **P4** because the response does not have at least two of the three correct x-values and presents incorrect reasoning.

In part C the response earned **P5** with the correct labeled value of 9 for $g(12)$ in line 1. The response earned **P6** with the correct labeled value of $-\frac{9}{2}\pi$ for $g(0)$ in line 2.

Question 4 (continued)

In part D the response earned **P7** with the equation $g'(x) = f(x) = 0$ in line 1. The response did not earn **P8** because a correct justification of an absolute minimum is not presented. The justification presented is not sufficient to establish that $x = 0$ is an absolute minimum on the given interval. The response earned **P9** for presenting the correct answer of $x = 0$ and a local argument that this is the location of a minimum.

Sample: 4C
Score: 3 (1-1-0-0-0-1-0-0-0)

The response earned 3 points: 2 points in part A, 0 points in part B, 1 point in part C, and 0 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 2. The equation $g'(8) = f(8)$ in line 3 would also have earned the point. The response earned **P2** with the boxed statement $g'(8) = 1$ in line 4.

In part B the response did not earn **P3** because $x = 0$ is presented as an answer in line 2. The response did not earn **P4**. An incorrect value of $x = 0$ is presented, making the response ineligible for **P3** or **P4**.

In part C the response did not earn **P5**. An incorrect value for $g(12)$ is presented. The response earned **P6** with the correct labeled value of $-\frac{9\pi}{2}$ for $g(0)$ in the boxed response given in the last line. The numeric expression $g(0) = -\frac{1}{2}\pi(3)^2$ would have earned the point in line 5 with no further simplification.

In part D the response did not earn **P7**. No evidence is presented that shows consideration of $g'(x) = 0$ or $f(x) = 0$. The response did not earn **P8**. A complete candidates test is not presented, as $g(0)$ is not considered. The response is eligible for **P9**. The response did not earn **P9** because the correct location of the minimum is not presented.